JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 23, No. 5, September–October 2000

Passivity-Based Robust Control with Application to Benchmark Active Controls Technology Wing

A. G. Kelkar
Kansas State University, Manhattan, Kansas 66506
and
S. M. Joshi
NASA Langley Research Center, Hampton, Virginia 23666

A passivity-based robust control design methodology is presented. A brief review of the stability results is given for passive linear and nonlinear systems followed by four different passification methods for rendering nonpassive systems passive. The robust control methodology via robust passification is demonstrated by application to the Benchmark Active Controls Technology subsonic wing. The controller design is shown to provide robust stability and satisfactory performance.

Introduction

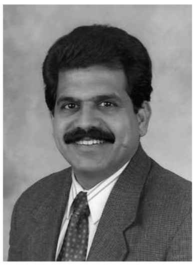
A NUMBER of aerospace as well as nonaerospace systems are characterized by highly elastic behavior. Examples of such systems include flexible aircraft, lightweight single-body and multibody spacecraft, and flexible robotic manipulators. Mathematical models of such systems consist of high-order oscillatory dynamics, large parametric uncertainties, and nonlinearities. The elastic modes usually have closely spaced natural frequencies and small inherent damping. For these reasons controller design for such systems is a difficult problem.

A significant subclass of elastic systems can be classified as naturally passive. Flexible space structures and multilink flexible manipulators, with collocated and compatible actuators and sensors, represent naturally passive systems. An interesting property of such systems is that they can be robustly stabilized by any strictly passive controller, despite unmodeled dynamics and parametric uncertainties. Therefore, passivity-based controllers have been found to be highly effective in controlling uncertain elastic systems.[1,2] However, many elastic systems are not inherently passive, and therefore passivity-based control techniques cannot be applied directly to such systems. Some examples of nonpassive systems include flexible aircraft, spacecraft, manipulators with noncollocated actuators and sensors, and unstable and/or nonminimum phase systems. One approach to make nonpassive systems amenable to passivity-based control is to passify them (i.e., render passive) using suitable compensation.[3] If the compensated system is robustly passive despite plant uncertainties, it can be robustly stabilized by any marginally strictly positive-real (MSPR) controller.[1]

In this paper a brief review of the stability results for passive linear and nonlinear systems is given followed by four different passification methods for systems that are not inherently passive. These methods can be used to robustly passify nonpassive systems. Once passified, such systems can be stabilized by any MSPR controller. The methodology of robust control via robust passification is applied to the Benchmark Active Controls Technology (BACT) subsonic wing.[4]

Passivity

The concept of passivity was first introduced in the network theory literature. For electrical networks passivity implies that any



Atul G. Kelkar received his Ph.D. degree in Mechanical Engineering from Old Dominion University, Norfolk, Virginia, in 1993. He held the position of Visiting Assistant Professor in the Aerospace Engineering Department at Old Dominion University from September 1993 to May 1994. He was National Research Council postdoctoral Research Associate at NASA Langley Research Center, Hampton, Virginia from 1994 to 1996. He is currently Associate Professor in the Mechanical and Nuclear Engineering Department at Kansas State University. Dr. Kelkar is a recipient of a National Science Foundation Career award. He is active in various professional societies. His publications include several conference and journal articles, and a book *Control of Nonlinear Multibody Flexible Space Structures* (London: Springer-Verlag, 1996; coauthored with S. M. Joshi). His research interests include robust control of aerospace systems, acoustic noise control, control of nonlinear systems, robotics, multidisciplinary optimization, flexible multibody dynamics, and neural networks.

Suresh M. Joshi received his Ph.D. degree in Electrical Engineering from Rensselaer Polytechnic Institute, Troy, NY, in 1973. He is presently Senior Scientist for Control Theory at NASA Langley Research Center in Hampton, Virginia. He also held visiting, adjunct, and research professor positions at various universities. His research interests include multivariable control theory, robust control, nonlinear systems, and applications to advanced aircraft and spacecraft. Dr. Joshi is a Fellow of the AIAA, the IEEE, and the ASME. He served on numerous editorial boards, technical committees, and conference organizing committees sponsored by these societies. His publications include several articles and two books *Control of Large Flexible Space Structures* (Berlin: Springer-Verlag, 1989) and *Control of Nonlinear Multibody Flexible Space Structures* (London: Springer-Verlag, 1996; coauthored with A. G. Kelkar). He is the recipient of a number of awards from NASA Langley Research Center, as well as the IEEE Control Systems Technology Award (1995).

Received 20 May 1999; revision received 2 February 2000; accepted for publication 3 March 2000.

single-port network consisting solely of resistors, capacitors, and inductors always dissipates energy. Similarly, for mechanical systems any spring-mass-damper system with nonnegative damping coefficients is passive.

We consider a class of nonlinear systems (denoted by Σ) affine in control, described by

$$\dot{x} = f(x) + g(x)u, \qquad y = h(x) + N(x)u \tag{1}$$

where the state vector $x \in \Re^n$, the input u and the output $y \in \mathcal{L}_{2e}^m$, $f(x) \in \Re^n$, $h(x) \in \Re^m$, $g(x) \in \Re^{n \times m}$, $N(x) \in \Re^{m \times m}$, and f, g, h, N smooth (i.e., infinitely differentiable, or C^∞) functions, and $f(0) = 0$, $h(0) = 0$. $\mathcal{L}_{2e}^m$ denotes an m-dimensional extended $\mathcal{L}_2$ space of all square integrable functions. We shall assume that the system Σ given by Eq. (1) is globally reachable and zero-state observable.[5]

Passivity can be defined in the input-output (IO) sense or in the internal sense.[6] The IO definition is more general and is applicable to a large class of systems including time-varying and infinite-dimensional systems.

IO Passivity

A system is said to be passive in the IO sense if there exists a constant β such that

$$\langle u, y\rangle_T + \beta \geq 0, \qquad \forall u \in \mathcal{L}_{2e}^m, \qquad \forall T \geq 0 \tag{2}$$

where $\langle \cdot, \cdot \rangle_T$ denotes truncated inner product.

Strict IO Passivity

A system is said to be strictly passive (or input strictly passive) in the IO sense if there exists a constant β and a constant $\epsilon > 0$ such that

$$\langle u, y\rangle_T + \beta \geq \epsilon \|u\|_T^2, \qquad \forall u \in \mathcal{L}_{2e}^m, \qquad \forall T \geq 0 \tag{3}$$

Internal passivity is usually defined for finite dimensional systems, as follows.

Internal Passivity

The system (Σ) is said to be internally passive if there exists a nonnegative storage function $E[\cdot]$ such that

$$\langle u, y\rangle_T \geq E[x(T)] - E[x(0)], \qquad \forall u \in \mathcal{L}_{2e}^m, \qquad \forall T \geq 0 \tag{4}$$

Strict (or input-strict) passivity in the internal sense is defined similarly.

The difference between IO passivity and internal passivity is that a nonnegative storage function (which is a function of the system's state vector) exists for the latter case.

Some passivity definitions for finite dimensional, linear, time-invariant (FDLTI) systems are given next. For such systems represented by a minimal realization, internal passivity and IO passivity are equivalent.

Passive FDLTI Systems

For FDLTI systems passivity is equivalent to positive realness of the transfer function. Let $G(s)$ denote an $m \times m$ matrix whose elements are proper rational functions of the complex variable s. $G(s)$ is said to be stable if its minimal realization is asymptotically stable. Let the conjugate-transpose of a complex matrix T be denoted by T^*.

Definition 1: An $m \times m$ rational matrix $G(s)$ is said to be positive real (PR) if 1) all elements of $G(s)$ are analytic in $Re(s) > 0$; 2) $G(s) + G^*(s) \geq 0$ in $Re(s) > 0$; or equivalently, a) poles on the imaginary axis are simple and have nonnegative-definite residues, and b) $G(j\omega) + G^*(j\omega) \geq 0$ for $\omega \in (-\infty, \infty)$.

For single-input, single-output (SISO) systems condition 2b) is equivalent to $Re[G(j\omega)] \geq 0$, or equivalently, the phase of $G(j\omega)$ remains between -90 and $+90$ deg. Given next are some definitions of strictly positive real (SPR) systems found in the literature. Definition 2, which represents the specialization of the general definition of strict passivity to stable LTI systems, is the strongest definition of strict positive realness.

Definition 2: An $m \times m$ stable rational matrix $G(s)$ is said to be strictly passive if there exists an $\epsilon > 0$ such that

$$G(j\omega) + G^*(j\omega) \geq \epsilon I \qquad \text{for} \qquad \omega \in (-\infty, \infty)$$

Strictly passive systems require the system to have a relative degree of zero, which makes this a very restrictive class of systems.

Definition 3 (Ref. 7): An $m \times m$ stable rational matrix $G(s)$ is said to be strictly positive real in the weak sense (weak SPR or WSPR) if

$$G(j\omega) + G^*(j\omega) > 0 \qquad \text{for} \qquad \omega \in (-\infty, \infty)$$

A stronger definition of SPR was given in Refs. 8 and 9, which requires certain additional conditions. Definitions 2 and 3 assume stable systems. The following definition, introduced in Ref. 10, allows the system to have poles on the imaginary axis.

Definition 4 (Ref. 10): An $m \times m$ rational matrix $G(s)$ is said to be MSPR if it is positive real, and

$$G(j\omega) + G^*(j\omega) > 0 \qquad \text{for} \qquad \omega \in (-\infty, \infty)$$

Definition 4 of MSPR differs from Definition 1 (PR) because the frequency domain inequality ($\geq$) has been replaced by the strict inequality ($>$). The difference between Definitions 3 and 4 is that the latter allows $G(s)$ to have poles on the imaginary axis. Thus Definition 4 gives the least restrictive class of SPR systems. If $G(s)$ is MSPR, it can be expressed as $G(s) = G_1(s) + G_2(s)$, where $G_2(s)$ is WSPR and all of the poles of $G_1(s)$ (in the Smith–McMillan sense) are purely imaginary.[10] Let $[A_i, B_i, C_i, D_i]$ denote n_i-order minimal realizations of $G_i(s)$, $(i = 1, 2)$.

State-Space Characterization of Passive Linear Systems

For FDLTI systems the well-known Kalman–Yakubovich lemma[11] gives the necessary and sufficient conditions for a system to be internally passive. In Ref. 7 the Kalman–Yakubovich lemma was extended to WSPR systems, and in Ref. 10 it was extended to MSPR systems. These extensions are given next. $\{[A, B, C, D]$ denotes an nth-order minimal realization of the $m \times m$ transfer function matrix $G(s)\}$.

Lemma 1 (Ref. 7): $G(s)$ is WSPR if and only if there exist real matrices: $P = P^T > 0$, $P \in \Re^{n \times n}$, $L \in \Re^{m \times n}$, $W \in \Re^{m \times m}$ such that

$$A^T P + PA = -L^T L \tag{5}$$

$$C = B^T P + W^T L \tag{6}$$

$$W^T W = D + D^T \tag{7}$$

where $[A, B, L, W]$ is minimal and $F(s) = W + L(sI - A)^{-1}B$ is minimum phase.

Proof: The proof can be found in Ref. 7.

Let $[A_2, B_2, C_2, D]$ denote an nth-order minimal realization of $G_2(s)$, the stable part of $G(s)$. The following lemma is an extension of the Kalman–Yakubovich lemma to the MSPR case.

Lemma 2 (Ref. 10): If $G(s)$ is MSPR, there exist real matrices: $P = P^T > 0$, $P \in \Re^{n \times n}$, $\mathcal{L} \in \Re^{m \times n_2}$, $W \in \Re^{m \times m}$ such that Eqs. (5–7) hold with

$$L = \left[0_{m \times n_1}, \mathcal{L}_{m \times n2}\right] \tag{8}$$

where $[A_2, B_2, \mathcal{L}, W]$ is minimal and $F(s) = W + L(sI - A)^{-1}B = W + \mathcal{L}(sI - A_2)^{-1}B_2$ is minimum phase.

Proof: Refer to Ref. 10.

In both WSPR and MSPR cases the storage function is given by $E(x) = \frac{1}{2}x^T Px$, where x is the state vector of a minimal realization of $G(s)$.

Stabilization of Passive Systems Using LTI Controllers

Stability analysis of the negative feedback interconnection of two passive systems has long been a topic of considerable interest. The best-known stability result for such systems is the Passivity Theorem, which states that the negative feedback interconnection of a passive system and a strictly passive system is finite gain (or $\mathcal{L}_2$) stable (e.g., see Ref. 11). ($\mathcal{L}_2$ stability is equivalent to asymptotic stability for minimal realizations of FDLTI systems). The requirement of strict passivity, however, is too restrictive. In particular, a strictly passive system requires a direct feedthrough term in the output equation (i.e., the relative degree has to be zero). It is, therefore, highly desirable to weaken the requirement of strict passivity.

To that effect, in the case of FDLTI systems the requirement of strict passivity was weakened in Refs. 7 and 10. It was shown in Ref. 10 that the negative interconnection of a PR system $[H(s)]$ and an MSPR system $[G(s)]$ is asymptotically stable if none of the $j\omega$-axis poles of $G(s)$ is a transmission zero of $H(s)$. The stability holds regardless of the presence of unmodeled dynamics and parametric uncertainty in the systems and is therefore robust. A special case occurs when $G(s)$ is WSPR, and for that case the resulting feedback loop is also asymptotically stable, as was stated in Ref. 7.

We shall next state the extensions of the stability results of Refs. 7 and 10 to nonlinear passive systems in a negative feedback loop with passive FDLTI systems.

Stability Result for Nonlinear Passive Systems

Consider the system shown in Fig. 1, which represents the negative feedback interconnection of a passive nonlinear system and an LTI system. The following theorem establishes global asymptotic stability of the feedback system under weaker conditions, i.e., when one of the systems is passive and the other is linear and WSPR or MSPR.

Theorem 1: Suppose in the system shown in Fig. 1, Σ is affine in control, internally passive, strongly zero-state observable, and has a radially unbounded storage function $E(x)$. Then the closed-loop system is globally asymptotically stable if 1) $G(s)$ is WSPR, or 2) if $G(s)$ is MSPR and Σ has the property that $\lim_{t\to\infty} y(t) = 0 \Rightarrow u(t) \in \mathcal{L}_2[0, \infty)$.

Proof 1: Please see Ref. 5.

The property given in condition 2) of the preceding theorem for Σ is equivalent to stating that the zero dynamics of the system is asymptotically stable, i.e., the system is minimum phase.

Stability Result for Linear Passive Systems

The following theorem gives conditions for stable interconnection of two passive linear systems.

Theorem 2 (Ref. 10): The negative feedback interconnection of $G(s)$ and $H(s)$ is stable if all of the following conditions are satisfied: 1) $G(s)$ is MSPR, 2) $H(s)$ is PR, and 3) none of the $j\omega$-axis poles of $G(s)$ is a transmission zero of $H(s)$.

An immediate consequence of the preceding result is that system is stable if either 1) $G(s)$ is WSPR and $H(s)$ is PR, or 2) both $G(s)$ and $H(s)$ are MSPR.

The importance of this result is that it holds regardless of the number of unmodeled modes or parameter errors, i.e., the stability is robust to modeling uncertainties. Most physical systems, however, are not inherently passive, and the stability results just given do not extend directly to these systems. One way to overcome this problem is to render such systems passive via suitable compensation and then make use of the preceding stability results for designing stabilizing controllers. The robustness of stability, however, now depends on the robustness of passification. If the passification is robust, stability is robust. Thus if nonpassive systems can be rendered robustly passive via appropriate compensation, it can be robustly stabilized by any MSPR controller, and the problem of robust control can be transformed into the problem of robust passification. In the next section we present some techniques for passification of nonpassive LTI systems. In most of the cases, the techniques presented are in the context of SISO systems; however, the methodology can be extended to multi-input multi-output (MIMO) systems as well.

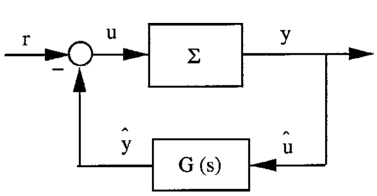


Fig. 1 Feedback interconnection.

Methods of Passification

Application of passivity-based methods to nonpassive systems has been addressed to a limited extent in the literature.[12–14] Much of this literature, however, has focused on almost strictly PR (ASPR) systems, which are systems that can be passified by sufficiently high constant-gain output feedback. Such systems represent a rather restrictive class because they have to be stable and minimum phase and cannot have a relative degree of more than one. In this section some passification methods are briefly presented for general nonpassive LTI systems.

Series Compensation

Consider the block diagram shown in Fig. 2a, where the plant $P(s)$ is nonpassive. The idea of series passification is to design a compensator $C_s(s)$ such that the compensated plant $P_c(s) = P(s)C_s(s)$ is PR. (For MIMO plants both right and left multipliers can be used). For stable minimum-phase systems with relative degree of zero or one, a proper series compensation can be obtained, which can render these systems PR.

For the SISO case the compensated system is passive if the phase of $P_c(s)$ remains within ±90 deg for all frequencies. In general, for plants that have real, imaginary, and complex poles and zeros, it is observed that the alternating pole-zero pattern for both real and imaginary parts results in positive realness. One of the techniques to obtain a series passification is to select a compensator whose pole-zero pattern along with plant's poles and zeros forms an alternating pole-zero pattern. An alternate method is to inspect the Bode diagram of $P(s)$ and to compute the phase required to make $P_c(s)$ PR. A more comprehensive discussion on ensuring positive realness in series passification can be found in Ref. 3.

Limitations of series compensation are that it cannot passify unstable or nonminimum phase plants and plants having repeated poles or zeros on the imaginary axis.

For systems with relative degree greater than one, the series passification would have to be improper. For physical realizability, however, the compensation must be made proper by placing high-frequency poles that are sufficiently outside the closed-loop bandwidth, resulting in bandlimited passivity.[3]

Another technique for series passification is the formulation of the problem in the linear matrix inequality (LMI) setting based on

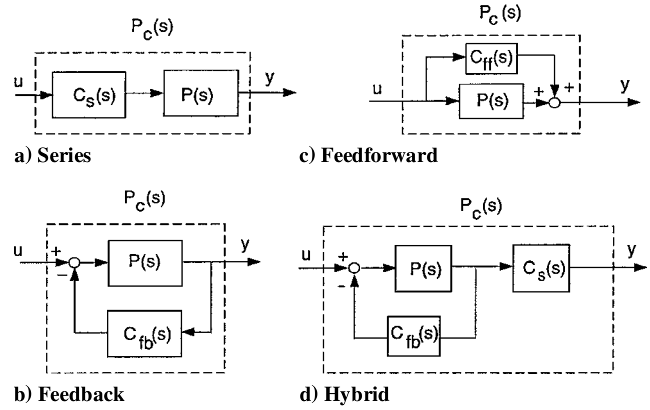


Fig. 2 Passification methods.

the Kalman–Yakubovich lemma. This approach can also be used for MIMO systems to yield left- and right-series compensators. Given next is the outline of the LMI passification method. Let the plant $P(s)$ have state-space realization:

$$P(s) \sim \begin{cases} \dot{x} = Ax + Bu' \\ y = Cx + Du' \end{cases} \tag{9}$$

and series passifier (Fig. 2a) be given by

$$C_s(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c u \\ u' = C_c x_c + D_c u \end{cases} \tag{10}$$

Then the compensated system has the following state-space realization:

$$P_c(s) \sim \begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u \\ y = \tilde{C}\tilde{x} + \tilde{D}u \end{cases} \tag{11}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} BD_c \\ B_c \end{bmatrix}$$
$$\tilde{C} = [C \quad DC_c], \qquad \tilde{D} = DD_c \tag{12}$$

For the compensated system $[P_c(s)]$ to be passive, we need $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ to satisfy the following inequality:

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} \\ \tilde{B}^T P & 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{D} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} U & W \\ W^T & V \end{bmatrix} \begin{bmatrix} \tilde{C} & \tilde{D} \\ 0 & I \end{bmatrix} < 0 \tag{13}$$

where $U = 0$, $V = 0$, and $W = -I$. This matrix inequlity is nonlinear in matrix variables A_c, B_c, C_c, D_c, and P; however, using techniques given in Refs. (15) and (16), Eq. (13) can be converted to an LMI, which can be solved using LMI toolbox in MATLAB® or other available LMI solution techniques.

Feedback Compensation

As already stated, certain nonpassive systems, such as unstable systems or systems having repeated poles/zeros on the imaginary axis, cannot be passified by series compensation alone. For such systems passification can sometimes be achieved by feedback compensation (Fig. 2b). For minimum-phase systems the condition for passification by feedback compensation can be easily derived as

$$Re[C_{\text{fb}}(j\omega)] \geq -\frac{Re[P(j\omega)]}{|P(j\omega)|^2}$$

Another example of systems where feedback compensation can be used for passification is ASPR systems,[12,13] which can be passified by a constant-gain output feedback. Such a compensation is also robust to uncertainties that satisfy certain boundedness conditions. Although this result guarantees that a constant feedback gain can passify such systems, the feedback gain required in some cases could be very large. The problem of dynamic feedback passification can also be formulated in the matrix inequality setting as shown next for a strictly proper plant.

Consider a plant

$$P(s) \sim \begin{cases} \dot{x} = Ax + Bu' \\ y = Cx \end{cases} \tag{14}$$

and controller (Fig. 2b)

$$C_{\text{fb}}(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c y \\ u_0 = C_c x_c + D_c y \\ u' = u - u_0 \end{cases} \tag{15}$$

Then the closed-loop system is given by

$$P_c(s) \sim \begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u \\ y = \tilde{C}\tilde{x} \end{cases} \tag{16}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A - BD_cC & -BC_c \\ B_cC & A_c \end{bmatrix}$$
$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \tilde{C} = [C \quad 0] \tag{17}$$

Now, for the closed-loop system to be passive, we need

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} \\ \tilde{B}^T P & 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} U & W \\ W^T & V \end{bmatrix} \begin{bmatrix} \tilde{C} & 0 \\ 0 & I \end{bmatrix} < 0 \tag{18}$$

where $U = 0$, $V = 0$, and $W = -I$. This nonlinear matrix inequality can be transformed into an LMI using suitable transformations, and a feedback passifier can be obtained.[15,16]

Feedforward Compensation

For certain systems, such as nonminimum phase systems or systems with high relative degree, the first three passification methods cannot be used. To passify such systems, a possible solution is to use feedforward compensation $C_{\text{ff}}(s)$,[12,13] as shown in Fig. 2c. If $C_{\text{ff}}(s)$ is a constant matrix, it has the effect of reducing the relative degree of the modified system to zero. In general, $C_{\text{ff}}(s)$ does not have to be a constant matrix but can be a proper transfer function. The condition for passification would be to obtain transfer function $C_{\text{ff}}(s)$ such that $Re[P(j\omega)] + Re[C_{\text{ff}}(j\omega)] \geq 0 \ \forall \omega \in (0, \infty)$. The LMI formulation to obtain such a passifier would involve the following: Consider a plant

$$G(s) \sim \begin{cases} \dot{x} = Ax + Bu \\ y_1 = Cx + Du \end{cases} \tag{19}$$

and feedforward compensator

$$C_{\text{ff}}(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_c u \\ y_2 = C_c x_c + D_c u \end{cases} \tag{20}$$

Then the compensated system is given by

$$G_c(s) \sim \begin{cases} \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u \\ y = \tilde{C}\tilde{x} + \tilde{D}u \end{cases} \tag{21}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}, \qquad \tilde{A} = \begin{bmatrix} A & 0 \\ 0 & A_c \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} B \\ B_c \end{bmatrix}$$
$$\tilde{C} = [C \quad C_c], \qquad \tilde{D} = D + D_c \tag{22}$$

Again, for the compensated system to be passive, we need, as before,

$$\begin{bmatrix} \tilde{A}^T P + P\tilde{A} & P\tilde{B} \\ \tilde{B}^T P & 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} & \tilde{D} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} U & W \\ W^T & V \end{bmatrix} \begin{bmatrix} \tilde{C} & \tilde{D} \\ 0 & I \end{bmatrix} < 0 \tag{23}$$

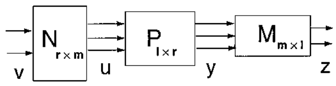


Fig. 3 Passification via sensor blending and control allocation.

where $U = 0$, $V = 0$, and $W = -I$. If $C_{ff}(s)$ is a constant matrix, feedforward passification problem can be formulated as an LMI. For a general case, however, Eq. (23) is a nonlinear matrix inequality that can be transformed into an LMI using the techniques given in Refs. 15 and 16.

Hybrid Compensation

In certain cases solely series, feedback, or feedforward passification may not be possible or desirable. For example, the feedback gain required to passify certain ASPR systems may be very large or, in some cases, the series passification alone may be very sensitive to plant variations. In such cases a hybrid compensator, which is a combination of series and feedback passification, may be more desirable. Figure 2d shows the configuration for hybrid compensation. In a number of example cases, it has been found that hybrid compensation significantly increases the robustness of series passification. An alternate configuration for hybrid passification would include $C_s(s)$ inside the feedback loop. For hybrid passification LMI techniques have not yet been developed.

Sensor Blending and Control Allocation

For certain systems a large number of redundant sensors and/or actuators may be available. For such systems it is sometimes possible to obtain passification by appropriately combining the sensor signals and/or distributing the control inputs to different actuators. Referring to Fig. 3, the problem is to find a sensor blending matrix M and a control allocation matrix N such that the system $MP(s)N$ is PR. Furthermore, M and N can be obtained to maximize the robustness of passification in the presence of parametric uncertainties. In Ref. 17, assuming the system matrices to be affine functions of uncertain parameters belonging to a convex polytopic region, an LMI-based approach was developed for robust passification via optimal sensor blending and control allocation.

Robustness of Passification

The stability robustness of the closed-loop system in the case of passified systems depends on the robustness of passification, i.e., it depends on how much uncertainty can be tolerated in the plant $[P(s)]$ before the compensated plant $[P_c(s)]$ loses its passivity property. In Ref. 18 frequency-domain sufficient conditions for robust passification were derived, which can be used to check the robustness of passification. Given next are the sufficient conditions (without proofs) for some selected combinations of the type of uncertainy and the type of passification. A more comprehensive list of conditions with their proofs can be found in Ref. 18. In the conditions given next, $\bar{\sigma}(\Delta)$ denotes the largest singular value of $\Delta(j\omega)$, and $\lambda_{\min}(T)$ denotes the smallest eigenvalue of $T(j\omega)$.

Series Passification

It is assumed here that a precompensator $C_s(s)$ is used for series passification. However, similar results can be obtained for postcompensator and for both pre- and postcompensators.

Additive Uncertainty

Theorem 2: Suppose a nonpassive plant $P(s)$ is passified by a series compensator $C_s(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty Δ is given by (Fig. 4a):

$$\bar{\sigma}(\Delta) < \frac{\lambda_{\min}\left(PC_s + C_s^* P^*\right)}{2\,\bar{\sigma}(C_s)}, \qquad \forall\omega \geq 0 \tag{24}$$

Feedback Uncertainty

Theorem 3: Suppose a nonpassive plant $P(s)$ is passified by a series compensator $C_s(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty Δ is given by (Fig. 4b):

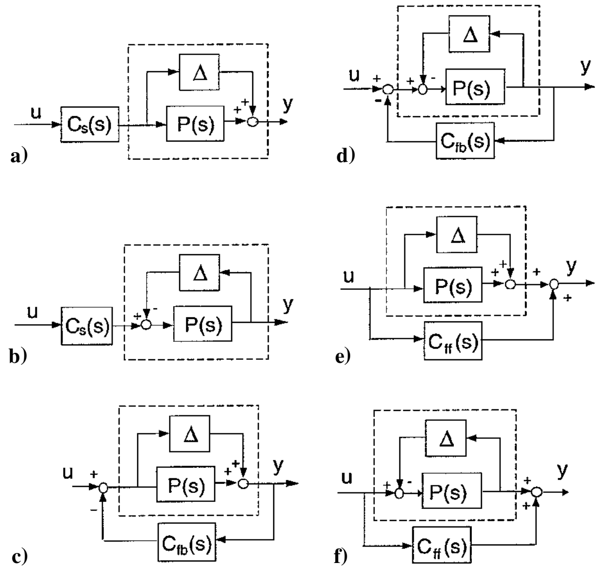


Fig. 4 Robustness of passification.

$$\bar{\sigma}(\Delta) < \frac{\lambda_{\min}\left(PC_s + C_s^* P^*\right)}{2\,\bar{\sigma}(PC_s)\bar{\sigma}(P)}, \qquad \forall\omega \geq 0 \tag{25}$$

Feedback Passification

Additive Uncertainty

Theorem 4: Suppose a nonpassive plant $P(s)$ is passified by a feedback compensator $C_{fb}(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty Δ is given by (Fig. 4c):

$$\bar{\sigma}(\Delta) < \left(-b + \sqrt{b^2 - 4ac}\right)\Big/2a, \qquad \forall\omega \geq 0 \tag{26}$$

where $a = \bar{\sigma}(C_{fb})$, $b = \bar{\sigma}(I + PC_{fb}) + \bar{\sigma}(PC_{fb})$, $c = -\lambda_{\min}(\bar{P} + \bar{P}^*)/2$, and $\bar{P} = P(I + PC_{fb})^*$, or Eq. (26) is satisfied with $a = \bar{\sigma}(\bar{P})\bar{\sigma}(C_{fb} + C_{fb}^*)$, $b = 2\bar{\sigma}(\tilde{P})[1 + \bar{\sigma}(\bar{P}C_{fb}^*)]$, $\tilde{P} = (I + PC_{fb})^{-1}$, $\bar{P} = \tilde{P}P$.

Feedback Uncertainty

Theorem 5: Suppose a nonpassive plant $P(s)$ is passified by a feedback compensator $C_{fb}(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty Δ is given by (Fig. 4d):

$$\bar{\sigma}(\Delta) < \frac{\lambda_{\min}(\bar{P} + \bar{P}^*)}{2\,[\bar{\sigma}(P)]^2}, \qquad \forall\omega \geq 0 \tag{27}$$

where $\bar{P} = P(I + PC_{fb})^*$ or if Eq. (27) is satisfied with $\bar{P} = (I + PC_{fb})^{-1}P$.

Feedforward Passification

Additive Uncertainty

Theorem 6: Suppose a nonpassive plant $P(s)$ is passified by a feedforward compensator $C_{ff}(s)$. Then a sufficient condition for robust passification in the presence of additive plant uncertainty Δ is given by (Fig. 4e):

$$\bar{\sigma}(\Delta) < \lambda_{\min}(\bar{P} + \bar{P}^*)/2, \qquad \forall\omega \geq 0 \tag{28}$$

where $\bar{P} = P + C_{ff}$.

Feedback Uncertainty

Theorem 7: Suppose a nonpassive plant $P(s)$ is passified by a feedforward compensator $C_{ff}(s)$. Then a sufficient condition for robust passification in the presence of feedback plant uncertainty Δ is given by (Fig. 4f):

$$\bar{\sigma}(\Delta) < \left(-b + \sqrt{b^2 - 4ac}\right)/2a, \qquad \forall\omega \geq 0 \tag{29}$$

where $a = \bar{\sigma}^2(P)\bar{\sigma}(C_{\mathrm{ff}} + C_{\mathrm{ff}}^*)$, $b = 2[\bar{\sigma}(P + C_{\mathrm{ff}}^*)\bar{\sigma}(P)]$, $c = -\lambda_{\min}(\bar{P} + \bar{P}^*)$, and $\bar{P} = P + C_{\mathrm{ff}}$.

The methodology of robust control via passification is demonstrated next by application to a fourth-order SISO BACT subsonic wing model.

Application to BACT Wing

The BACT wing is the wind-tunnel model of an experimental subsonic airfoil built at NASA Langley Research Center for validating mathematical models and control laws for aeroelastic systems. A schematic of BACT wing can be found in Figs. 1 and 2 of Ref. 19 (also in Ref. 4). The BACT wing is a rectangular wing with a NACA 0012 airfoil section and is equipped with pressure transducers and three hydraulically actuated control surfaces, namely, trailing edge and upper and lower spoilers. The model was mounted on a pitch and plunge apparatus in the NASA Transonic Dynamics Tunnel to test flutter suppression control law designs. Linear accelerometers are located at each corner of the wing. For the BACT system the transonic flutter occurs for a range of Mach numbers from approximately 0.7 to 0.85 (Ref. 20). For a Mach number of 0.77, the flutter frequency is approximately 26 rad/s (i.e., about 4 Hz), and the dynamic pressure is approximately 150 psf. In this paper a linearized SISO fourth-order model of the system at Mach 0.77 and the dynamic pressure of 150 psf is used for the controller design studies. In Ref. 20 a similar model was used for control law design studies,

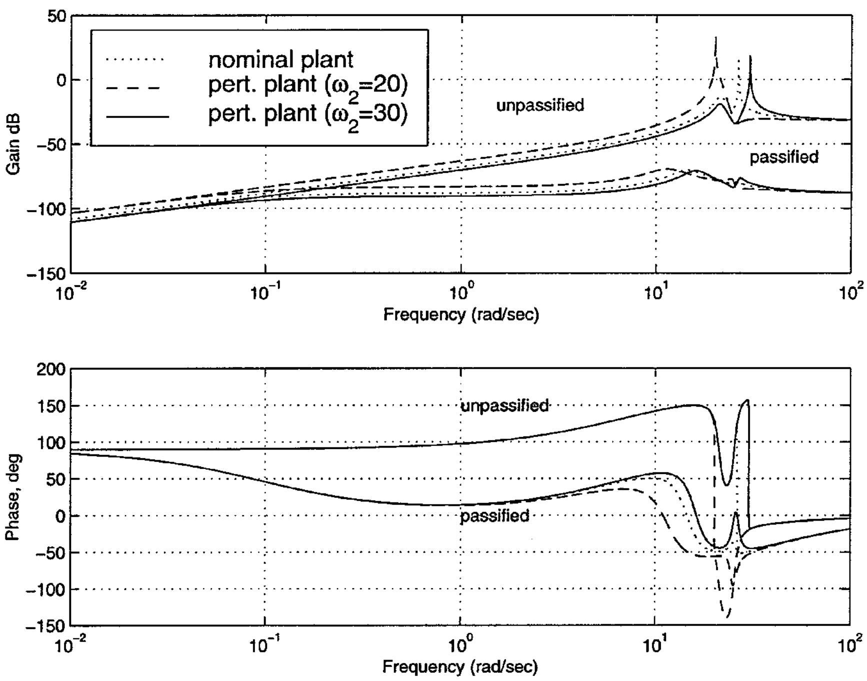


Fig. 5 Bode plot comparison for open-loop and passified system.

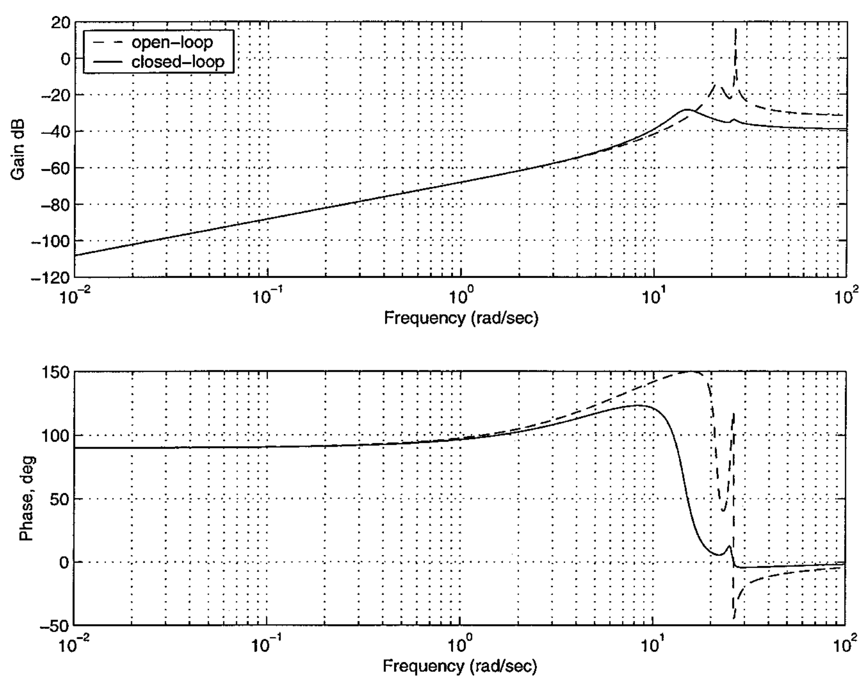


Fig. 6 Open- and closed-loop Bode plots for nominal plant.

whereas in Ref. 19 a slightly different model at higher dynamic pressure (approx. 225 psf) was used. In both of these, the plant output was taken to be the difference of trailing- and leading-edge accelerometer readings. The control law design in Ref. 20 was primarily based on the multivariable design model with two inputs and two outputs whereas in Ref. 19, various flutter suppression control laws were designed for the SISO design model using the same output but only one of the control surfaces for the input. For the design model used in this paper, the input is the upper spoiler control, and the output is the trailing-edge accelerometer. The nominal transfer function for this IO pair is given by

$$P(s) = \frac{0.0253s(s^2 + 2.28s + 25.29^2)(s + 7.47)}{(s^2 + 2.26s + 21.17^2)(s^2 + 0.023s + 26.23^2)} \tag{30}$$

The design model used has two aeroelastic modes with natural frequencies (nominal values) $\omega_1 = 21$ rad/s and $\omega_2 = 26$ rad/s with damping ratios $\zeta_1 = 0.053$ and $\zeta_2 = 0.0004$, respectively. The open-loop plant is not passive and is characterized by sharp peaks in the frequency response with a phase angle of nearly 150 deg at certain frequencies. The control objective is to suppress the flutter and maintain stability over the entire range of operating conditions using a single, low-order controller that is easy to implement. The first step in the controller design is to passify robustly the system in the presence of a given range of uncertainties in ω_1, ω_2, ζ_1, and ζ_2. Because of the significant damping that is present in ω_1, the Bode plot is not appreciably affected by the uncertainty in ω_1, although it is quite sensitive to the variation in ω_2. A series compensation was first attempted for passification of the system. However, it was not possible to robustly passify the system using series compensation

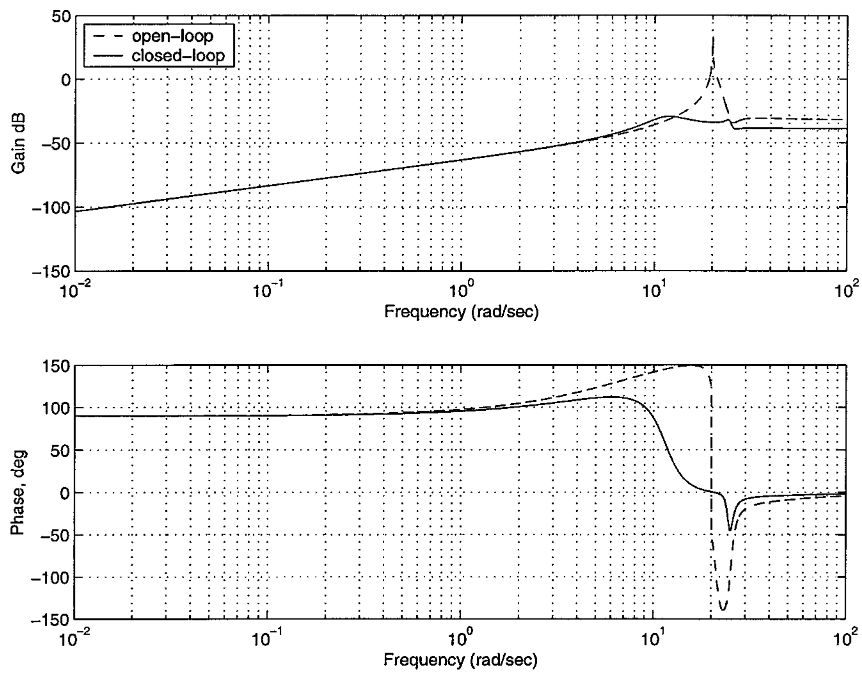


Fig. 7 Open- and closed-loop Bode plots for perturbed plant ($\omega_2 = 20$).

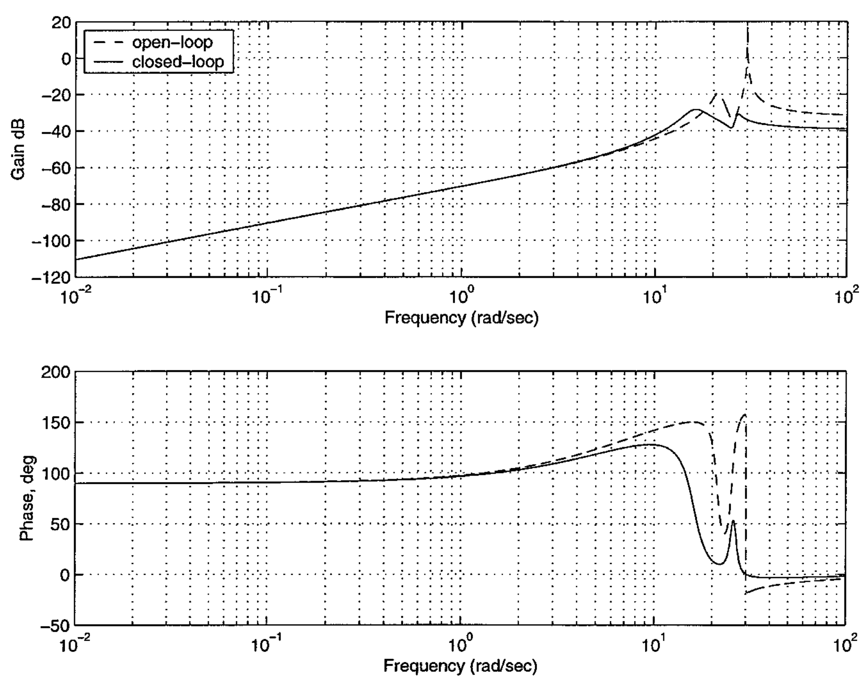


Fig. 8 Open- and closed-loop Bode plot for perturbed plant ($\omega_2 = 30$).

alone; therefore, a hybrid feedback/series compensation was designed. First, the loop was closed with constant output feedback gain $\gamma = 50$. The phase of the resulting system, however, exceeds 90 deg; therefore, after closing the loop the following additional series compensation was chosen to passify the system:

$$G_c(s) = \frac{(s + 30)}{300(s + 0.1)} \tag{31}$$

The resulting system was robustly passive for $20 \leq \omega_2 \leq 30$ (Fig. 5). A large variation of ω_1 did not affect passivity. The passivity was also preserved for a four-fold reduction in ζ values. Having robustly passified the system, the next step is to design a passivity-based outer-loop controller. The simplest MSPR controller that accomplished the objective was a constant gain output feedback controller $\gamma_{\mathrm{pr}} = 100$. Root locus analysis revealed a three-fold increase in the damping ratio (to $\zeta_1 = 0.16$) for the first mode and a 100-fold increase (to $\zeta_2 = 0.037$) for the second mode. The closed-loop frequency response (which indicates vibration suppression) is reasonably flat and has no sharp peaks, as shown in Fig. 6. When ω_2 was perturbed in the range $20 \leq \omega_2 \leq 30$, the closed-loop Bode plot was almost unchanged, whereas the open-loop Bode plot showed peaks that were sharper and taller than those for the nominal case (Figs. 7 and 8). Figures 5–8 show that the closed-loop response is fairly flat compared to the open-loop response for the nominal as well as the perturbed plant conditions. The reduction in magnitude peaks ranges from 44–63 dB. This indicates satisfactory vibration suppression over all operating conditions. In addition to the perturbations in ω_2, the damping ratios ζ_i were also changed to check the robustness of the design. Figure 9 shows the comparison of the open-loop and closed-loop Bode plots for the case when a four-fold reduction in the damping ratio was assumed for both the modes, with ζ_1 perturbed from 0.053 to 0.013 and ζ_2 perturbed from 0.0004 to 0.0001. Figures 10 and 11 show the Bode plots when there

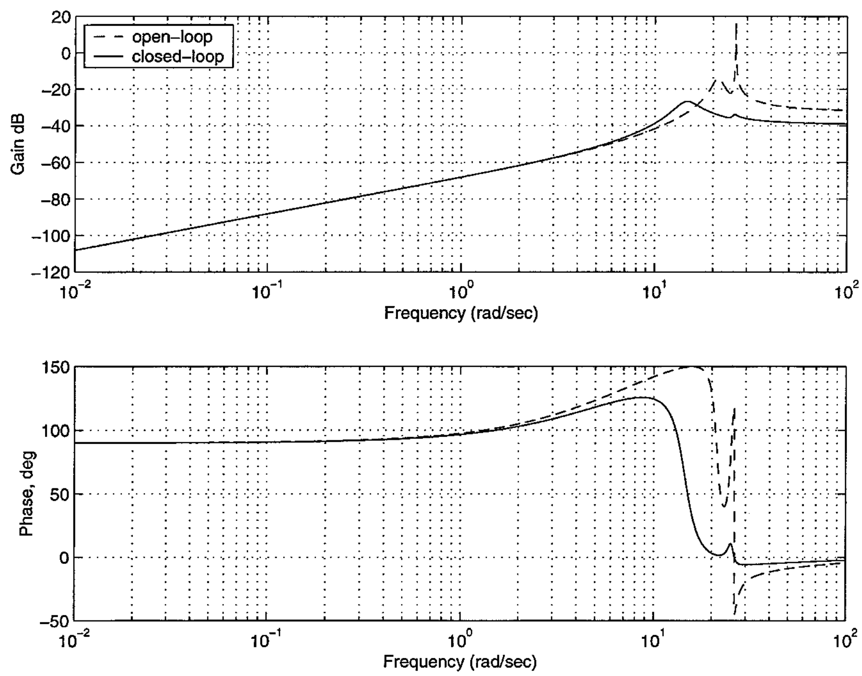


Fig. 9 Open- and closed-loop Bode plots for perturbation in ζ only.

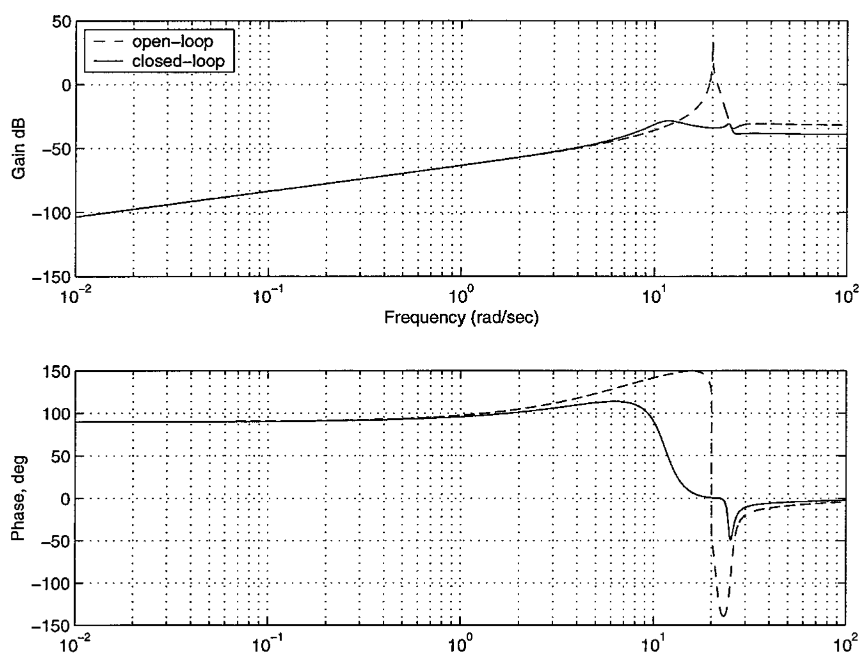


Fig. 10 Open- and closed-loop Bode plots for ζ_1 = 0.013, ζ_2 = 0.0001, and ω_2 = 20 rad/s.

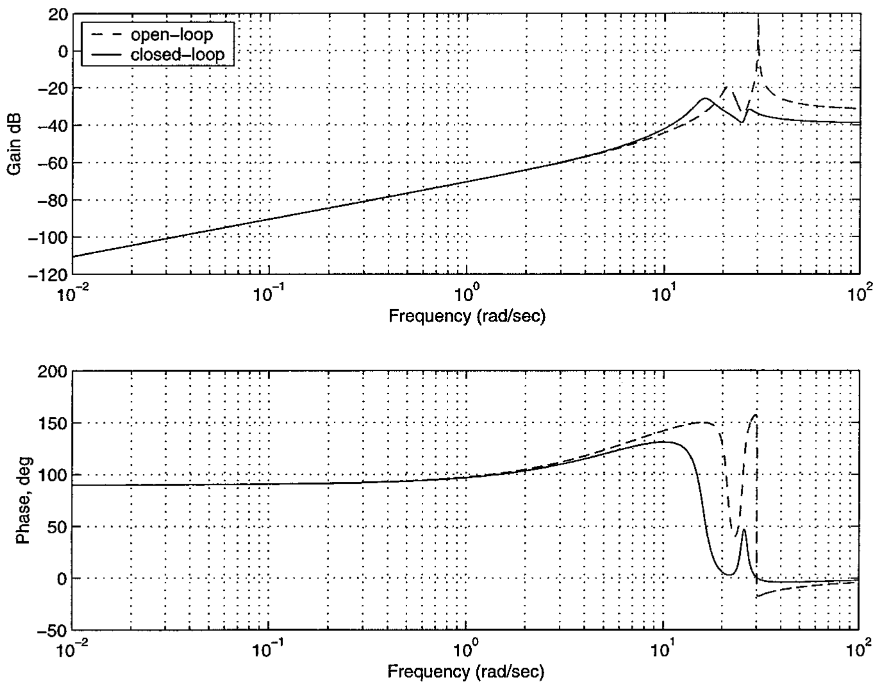


Fig. 11 Open- and closed-loop Bode plots for $\zeta_1 = 0.013$, $\zeta_2 = 0.0001$, and $\omega_2 = 30$ rad/s.

are simultaneous perturbations in ω_2, ζ_1, and ζ_2. As seen from these plots, the design is robust to the simultaneous perturbations as well.

The transfer function of the complete compensator (including the hybrid passifier and the constant-gain output feedback) is $C(s) = \gamma + \gamma_{pr} G_c(s)$, which has a dc gain of about 43 dB and phase variation between 0 and -30 deg. The controller dc gain compares well with that of the controllers designed in Refs. 19 and 20. The controller has infinite gain margin for nominal as well as perturbed plant conditions, and its phase margin varies between 31–37 deg. Another measure for robustness, the minimum (over ω) of $|(1 + P(j\omega)C(j\omega)|$, was also computed for nominal as well as perturbed plant conditions. These values range from 0.52 to 0.63, which are similar to those obtained in Ref. 19. This indicates that the control law design is very robust. The important advantage of this controller is its simplicity and low order. It is a simple first-order controller, which provides the desired robust stability and performance and is easy to implement. The robust control methods such as H_∞ and μ-synthesis yield controllers have at least the same order as the plant. (Usually, the controller order for such controllers is much higher than the plant because of the weighting functions.) The controller designed, using a passivity approach, provides infinite gain margin, has low order, and its simple structure makes it easy to implement.

The sufficient conditions (Eqs. 24–29) could not be used directly for the BACT problem because hybrid passification was used, for which robustness conditions are not available to date. However, for this SISO system, it was possible to check the robustness of passification directly by applying worst-case perturbations to the plant and examining the phase plot of the compensated plant.

Conclusions

A passivity-based controller design approach was considered for nonpassive linear, time-invariant systems. The approach consists of robust passification (i.e., rendering passive by compensation) of the system and subsequent design of a positive real controller, which then provides robust stability in spite of modeling errors and parametric uncertainties. Different methods of passification were given, and the controller design approach was applied to the BACT wing model. The controller design process consists of designing a compensator to passify the open-loop system followed by the design of an outer-loop passive controller. The hybrid passification was designed to be robust to changes in the elastic mode frequencies and damping ratios. In addition, the controller has infinite gain margin and a satisfactory phase margin. In summary, a hybrid passifier along with a constant-gain outer-loop passive controller provides both robust stability and satisfactory vibration supression for the BACT wing.

Acknowledgments

The first author acknowledges the support of NASA Langley Research Center (Grant NCC 1-226) and National Science Foundation (Grant CMS:9703092).

References

[1]Kelkar, A. G., and Joshi, S. M., *Control of Nonlinear Multibody Flexible Space Structures*, Vol. 221, Lecture Notes in Control and Information Sciences, Springer–Verlag, London, 1996.

[2]Joshi, S. M., *Control of Large Flexible Space Structures.* Vol. 131, Lecture Notes in Control and Information Sciences, Springer–Verlag, Berlin, 1989.

[3]Kelkar, A. G., and Joshi, S. M., "Robust Control of Non-Passive Systems via Passification," *Proceedings of the 1997 American Control Conference*, Vol. 5, 1997, pp. 2657–2661.

[4]Waszak, M. R., "Modeling the Benchmark Active Control Technology Wind-Tunnel Model for Application to Flutter Suppression," AIAA Paper 96-3437, July 1996.

[5]Joshi, S. M., and Kelkar, A. G., "Robust Stabilization of a Class of Passive Nonlinear Systems," NASA TM-110287, Oct. 1996.

[6]Hill, D. J., "Nonlinear Controller Design Using Passivity and Small Gain Techniques," Workshop Notes IEEE Conf. on Decision and Control, Dec. 1994.

[7]Lozano-Leal, R., and Joshi, S. M., "Strictly Positive Real Functions Revisited," *IEEE Transactions on Automatic Control*, Vol. 35, Nov. 1990, pp. 1243–1245.

[8]Taylor, J. H., "Strictly Positive Real Functions and Lefschetz-Kalman-Yakubovich (LKY) Lemma," *IEEE Transactions on Circuits Systems*, March 1974, pp. 310, 311.

[9]Ioannou, P., and Tao, G., "Frequency-Domain Conditions for Strictly Positive Real Functions," *IEEE Transactions on Automatic Control*, Vol. 32, Jan. 1987, pp. 53, 54.

[10]Joshi, S. M., and Gupta, S., "On a Class of Marginally Stable Positive Real Systems," *IEEE Transactions on Automatic Control*, Vol. 41, No. 1, Jan. 1996, pp. 152–155.

[11]Vidyasagar, M., *Nonlinear Systems Analysis*, Prentice–Hall, Upper Saddle River, NJ, 1993.

[12]Bar-Kana, I., "Parallel Feedforward and Simplified Adaptive Control," *International Journal of Adaptive Control and Signal Processing*, Vol. 1, No. 2, 1987, pp. 95–109.

[13]Kaufman, H., Bar-Kana, I., and Sobel, K., *Direct Adaptive Control Algorithms: Theory and Applications*, Springer–Verlag, New York, 1994.

[14]Sun, W., Khargonekar, P. P., and Shim, D., "Solution to the Positive Real Control Problem for Linear Time-Invariant Systems," *IEEE Transactions on Automatic Control*, Vol. 39, No. 10, 1994, pp. 2034–204.

[15]Kelkar, A. G., Mao, Y., and Joshi, S. M., "LMI-Based Passification for Control of Non-Passive Systems," *Proceedings of the 2000 American Control Conference*, Chicago, June 2000, pp. 1271–1275.

[16]Scherer, C., Gahinet, P., and Chilali, M., "Multiobjective Output-Feedback Control via LMI Optimization," *IEEE Transactions on Automatic Control*, Vol. 42, No. 7, 1997, pp. 896–911.

[17]Joshi, S. M., and Kelkar, A. G., "Robust Passification via Optimal Sensor Blending and Control Allocation," *Proceedings of the 1999 American Control Conference*, 1999, pp. 278–282.

[18]Kelkar, A. G., and Joshi, S. M., "Robust Passification and Control of Non-Passive Systems," *Proceedings of the 1998 American Control Conference*, 1998, pp. 3133–3137.

[19]Mukhopadhyay, V., "Transonic Flutter Supression Control Law Design and Wind Tunnel Test Results," AIAA Paper 99-1396, 1999.

[20]Waszak, M. R., "Robust Multivariable Flutter Suppression for the Benchmark Active Control Technology (BACT) Wind-Tunnel Model," *Proceedings of the 11th Symposium on Structural Dynamics and Control*, May 12–14, 1997, pp. 267–279.